APPENDIX 1

Introduction

Constitutive equations were derived to properly process the accelerometer output data into acceptable forms for testing for both mathematical reliability and biomechanical engineering validity related to occupant protection in rollovers. The equations used in this study describe a deformable body that is undergoing general translational and rotation motion as well as deformation. Six degrees of freedom are required for general translation and rotation and typically utilize a large number of degrees of freedom to model deformation. However, because we are concerned, at the present time, with processing data from accelerometers fixed to various points on the vehicle, we did not, for the purposes of this study, need to consider the number of degrees of freedom used to model the deformation. We only needed to model the part of the acceleration due to the deformation appropriately. Hence, we developed kinematic equations for the relative motion of each sensor with respect to a common point for which we know the acceleration. Since these equations contain the angular velocity and angular acceleration of the vehicle, we considered the problem of determining the rotational motion from the available data.

Kinematics

In Figure 1, the $O_X Y_Z$ system is an “Earth-fixed” coordinate system which is fixed in location and orientation. The vector $\vec{R}_C$ from O to C, the original center of mass of the vehicle, and the vector $\vec{r}_7$ locates a sensor denoted as “7” in the earth-fixed coordinate system. In the rollover tests, sensor S7 is a two-axis accelerometer at the B-pillar on the driver’s side. The acceleration measured by sensor S7 is equal to the acceleration of the center of mass, C, of the vehicle, plus the acceleration of S7 relative to C, i.e., the acceleration due to rotation of the vehicle about C and the acceleration due to localized rail/pillar deformation. This may be expressed mathematically as

$$\vec{A}_{S7} = \vec{A}_C + \vec{A}_{S7/C}$$  \hspace{1cm} (1)

where $\vec{A}_{S7}$, $\vec{A}_C$, and $\vec{A}_{S7/C}$ are the accelerations of S7, C, and S7 with respect to C, respectively. It follows from (1) that the acceleration of S7 with respect to C is

$$\vec{A}_{S7/C} = \vec{A}_{S7} - \vec{A}_C$$  \hspace{1cm} (2)
Figure 1

Since part of the acceleration of the sensors with respect to the center of gravity is due to the rotation of the vehicle, the angular velocity and angular acceleration of the vehicle-fixed axes must be used to compute the total acceleration. Two methods were utilized in this investigation to determine the angular velocity. One method uses accelerometer data from several sensors and the second uses a combination of accelerometer data and roll rate sensor data. If very good estimates of the angular velocity can be obtained, then the vehicle’s attitude may be obtained by numerical integration of appropriate kinematic equations. Also, the parts of the accelerations of the sensors with respect to the center of gravity that are due to the rotation of the vehicle may be removed from equations like Eq. (2). Then the part of the acceleration due only to deformation may be integrated to get deformation rates and displacements. Because each sensor has its own coordinate system, if the deformations are extreme (e.g. significant roof crush into the occupant survival space), then some method must be devised to account for the relative rotations of individual sensors.

If the components of $\ddot{A}_{s7/C}$ in the CXYZ system are used to calculate velocity and position, then the results should not contain the principal terms due to the translation of the center of mass of the vehicle. However, the rotation of the vehicle must still be properly included.

By definition, the acceleration $\ddot{A}_{s7/C}$ is the second time derivative of $r_7$. The latter may be written (See, for example, Meriam, 1971) as
\[ \ddot{A}_{S7/C} = \delta^2 \ddot{r}_7 / \delta t^2 + 2 \ddot{\omega} \times \delta \ddot{r}_7 / \delta t + \ddot{\omega} \times (\ddot{\omega} \times \ddot{r}_7) + \dddot{\omega} \times \dddot{r}_7 \] (3)

where \( \ddot{\omega} \) is the angular velocity of the CXYZ coordinate system and the derivative of a vector \( \ddot{r}_j \) indicates the time derivative of that vector as seen in the rotating (vehicle-fixed) system CXYZ. The quantity \( \delta^2 \ddot{r}_j / \delta t^2 \) is the relative acceleration (acceleration as viewed by an occupant of the vehicle as he/she rotates with the vehicle-fixed CXYZ system) due to the deformation of the vehicle's structure at point P7 to which the sensor S7 is attached. Now, \( \ddot{r}_7 \) may be written as

\[ \ddot{r}_7 = \ddot{r}_{70} + \delta \ddot{r}_7 \] (4)

where \( \ddot{r}_{70} = X_{70} \hat{I} + Y_{70} \hat{J} + Z_{70} \hat{K} \) is the position vector of point P7 on the driver's roof rail/B-pillar in the vehicle-fixed CXYZ system when there is no deformation of the roof rail and \( \delta \ddot{r}_7 = \delta X_7 \hat{I} + \delta Y_7 \hat{J} + \delta Z_7 \hat{K} \) is the displacement of P7 due to local deformation (“crush”) of the roof rail/B-pillar.

In most structures, under elastic deformation conditions, the displacements \( \delta X_7, \delta Y_7, \text{ and } \delta Z_7 \) are related by the fundamental mode shapes of the structure. In the present case of a compact SUV, the deformation is a combination of elastic and plastic, dynamic and residual deformation types. Since the analytical description of such deformations is beyond the scope of this investigation, an approach in which the displacements \( \delta X_7, \delta Y_7, \text{ and } \delta Z_7 \) are first considered to be independent and then the rotation of the sensor is estimated on the basis of the translation of the sensor appears to be reasonable.

Thus, assuming that there is little rotation of the vehicle's structure at P7 due to deformation, we may write

\[ \delta^2 \ddot{r}_7 / \delta t^2 = \delta \ddot{X}_7 \hat{I} + \delta \ddot{Y}_7 \hat{J} + \delta \ddot{Z}_7 \hat{K} \] (5)

If the angular velocity \( \ddot{\omega} \) and, hence, the angular acceleration \( \dddot{\omega} \), as functions of time are available from an angular velocity transducer and, if S7 and S11 are triaxial accelerometers, then estimates of \( \delta X_7, \delta Y_7, \text{ and } \delta Z_7 \) may be obtained from

\[ \delta^2 \ddot{r}_7 / \delta t^2 = -2 \dddot{\omega} \times \dddot{r}_7 / \delta t - \ddot{\omega} \times (\dddot{\omega} \times \dddot{r}_7) - \dddot{\omega} \times \dddot{r}_7 + \dddot{\omega} \times \dddot{r}_7 + \dddot{A}_{S7/C} \] (6)

Or, in matrix form for sensor Sj,

\[ \dddot{r}_j = -2 \dddot{\omega} \times \dddot{r}_j - \ddot{\omega} \times (\dddot{\omega} \times \dddot{r}_j) - \dddot{\omega} \times \dddot{r}_j + \dddot{A}_{Sj} - A_C \] (7)

where
In Eq. (7), \( \mathbf{a}_c \) contains the components of the acceleration of the center of mass measured in the CXYZ system, while \( \mathbf{a}_{sj} \) contains the components of the acceleration of sensor \( S_j \) measured in the \( S_jxyz \) system. If there is significant relative rotation of these coordinate systems then we must, of course, consider it.

### Estimation of Angular Velocity and Attitude Angles without and with Roll Sensor Data (Autoliv Test B190042)

The data taken during Autoliv’s Test B190042 include three-dimensional acceleration data from an accelerometer at the Visteon Fleet Roll Sensor, Autoliv Reference No. S1 [Ref. 1, page 18]. This additional data provides the relative acceleration of a third point in the vehicle that can be used to estimate the angular velocity and attitude of the vehicle. In the “vehicle-fixed” coordinate system, the sensor locations are identified by the respective position vectors of S1 (Visteon Fleet Roll Sensor, Rvis), S4 (Driver Rocker Panel Accelerometer at the B-pillar, Rdrpbp), and S9 (Passenger Rocker Panel Accelerometer at the B-pillar, Rprpbp), which are

\[
\mathbf{R}_1 = 1635.00 \mathbf{i}_G - 59.90 \mathbf{j}_G + 961.00 \mathbf{k}_G \text{ mm} \tag{8a} \\
\mathbf{R}_4 = 2802.90 \mathbf{i}_G - 768.10 \mathbf{j}_G + 762.30 \mathbf{k}_G \text{ mm} \tag{8b} \\
\mathbf{R}_9 = 2833.70 \mathbf{i}_G - 716.40 \mathbf{j}_G + 750.70 \mathbf{k}_G \text{ mm} \tag{8c} \\
\]

Similarly, the global position vector of the center of gravity is

\[
\mathbf{R}_{C.G.} = 2073.10 \mathbf{i}_G - 24.50 \mathbf{j}_G + 975.00 \mathbf{k}_G \text{ mm} \tag{9} \\
\]
These points are shown in Figure 2.

![Figure 2
Six Sensor Locations for Autoliv Test B190042](image)

The positions of the accelerometers with respect to the C.G. are

\[
\vec{\mathbf{r}}_1 = -438.10 \mathbf{i} - 35.40 \mathbf{j} + 14.00 \mathbf{k} \text{ mm} \quad (10a)
\]

\[
\vec{\mathbf{r}}_4 = 729.80 \mathbf{i} - 743.60 \mathbf{j} - 212.70 \mathbf{k} \text{ mm} \quad (10b)
\]

\[
\vec{\mathbf{r}}_9 = 760.60 \mathbf{i} + 740.90 \mathbf{j} - 224.30 \mathbf{k} \text{ mm} \quad (10c)
\]

We can use the matrix form of the relative accelerations from the three accelerometers,

\[
\ddot{\mathbf{r}}_j = -2 \mathbf{\ddot{\omega}} \dot{\mathbf{r}}_j - \mathbf{\ddot{\omega}} \mathbf{\ddot{\omega}} \mathbf{r}_j + \mathbf{\ddot{r}}_j \mathbf{\ddot{\omega}} + A_{Sj} - A_C, j = 1, 4, 9 \quad (11)
\]

and assume that for the time of interest, the structural deformation is zero at each of these accelerometers to get

\[
- \ddot{\mathbf{r}}_j \mathbf{\ddot{\omega}} = - \mathbf{\ddot{\omega}} \mathbf{\ddot{\omega}} \mathbf{r}_j + A_{Sj/C}, \ j = 1, 4, 9 \quad (12)
\]
We have nine equations from which we can find $\dot{\omega}$, but because of the skew-symmetry of the, $\tilde{r}_j$ we have only six independent ones. Still these are more than we need to find $\dot{\omega}$, so we use a weighted least squares approach. (Brogan, 1974) We pre-multiply the jth equation by $\tilde{r}_j W_j$, where $W_j$ is a constant, diagonal, 3x3 weighting matrix, and add the results to get

$$ I \dot{\omega} = -\tilde{\omega} I \tilde{\omega} + \tilde{r}_1 W_1 A_{SI/C} + \tilde{r}_4 W_4 A_{S4/C} + \tilde{r}_9 W_9 A_{S9/C} $$

(13)

In Eq. (13),

$$ I = -\tilde{r}_1 W_1 \tilde{r}_1 - \tilde{r}_4 W_4 \tilde{r}_4 - \tilde{r}_9 W_9 \tilde{r}_9 $$

(14)

is analogous to the inertia matrix of a rigid body and the sensor terms are analogous to torques.

By using the weighting matrices

$$ W_1 = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} $$

(15a)

and

$$ W_4 = W_7 = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} $$

(15b)

we obtained the time histories for the Test B190042 angular velocity components and Euler angles of the vehicle shown in Figures 3 and 4, respectively. The weights are somewhat arbitrary, but their sum should be 1. The Visteon accelerometer output was weighted heavier than that of the other two sensors because such weighting gives better results for pitch and yaw.
Figure 3
Estimated Angular Velocity Components – No Roll Rate Sensor
(Autoliv Test B190042)

Figure 4
Estimated Euler Angles - No Roll Rate Sensor Data
Note that because the vehicle Z-axis is initially directed upward and the X-axis is rearward, a positive pitch angle puts the nose of the vehicle higher and a positive yaw angle means that the nose of the vehicle has rotated towards the left from the viewpoint of a driver. A positive roll angle is initially a rotation of the driver's side of the vehicle toward the ground.

**Attitude Estimates Obtained by Including the Systron Roll Rate Sensor Data**

The data collected during Test B190042 included the output from the Systron Donner Roll Rate Sensor. Since this “roll rate data” is actually the angular velocity about the X-axis of the vehicle, it may be used as the X-component of angular velocity in our estimate of angular velocity and the other two components may be obtained as indicated above. Figures 5 and 6 show the resulting time histories of the angular velocity components and the Euler angles. Note that the agreement between the time histories of the X-components of the angular velocity shown in Figure 3 and Figure 5 is very good except for the oscillatory content in $\omega_x$ in Figure 5. Because the rate data was used directly to obtain Figure 5, the $\omega_x$ time history shown there still has considerable oscillatory content. On the other hand, the $\omega_x$ plot in Figure 3, which was obtained by integrating the accelerometer outputs after they have been filtered (60 Hz), has little high frequency content.

![Figure 5](image-url)

**Figure 5**

*Angular Velocity Components - Including Systron Roll Rate Data*
The results for the Euler angles that were obtained using the four accelerometers (C.G., DRPBP, PRPBP, and VISTEON) and the Systron Donner Roll Rate Sensor are presented in Figure 6. Note that the Systron Donner sensor measures the angular velocity about the X-axis, not the time rate of change of the roll angle $\phi$ (Phi). The attitude angle time histories obtained with the roll rate sensor data and without that data are very similar.

![Figure 6](image)

**Figure 6**

Estimated Euler Angles - Including Systron Roll Rate Sensor Data.

**Analysis of Data from B190043**

Although SAE recommended procedure provides that the vehicle's angular velocity be measured, it appears that in at least one Autoliv test (B190043) no angular velocity data was collected. Also, the accelerometer S7 provided only Y- and Z-accelerations in all four Autoliv tests. The angular velocity, however, may be estimated for those tests in which no angular velocity measurements were made, by using the procedure used for B190042. The vehicle CG accelerometer (S11) and any two triaxial accelerometers that are positioned such that the three are not collinear may be used. Figure 7 presents such an estimate obtained using sensors S4 and S9. These two are not collinear with C. The estimates of angular velocity components are similar to those in the Controlled Rollover Impact System (CRIS) study. (Carter et.al., 2002) However, shortly after 500
ms some large changes in accelerations occur of the points which S7 and S9 are attached. This limits the accuracy of the estimated angular velocity after 500 ms. Fortunately, there is another way to estimate the dynamic crush using Eq. (7).

![Graph of Angular Velocity Estimated from Accelerometer Data](image)

**Figure 7**
Angular Velocity Estimated from Accelerometer Data
(Autoliv Test B190043)

The terms due to angular velocity in Eq. (16) are fairly constant just before the acceleration of the sensor in the B-pillar becomes very large. Thus, if the value of the right-hand side of Eq. (16) at time $t_{start}$ before the large acceleration pulse is used as the part of $\frac{\delta^2 \mathbf{r}_7}{\delta t^2}$ not due to the crushing, then the part of $\frac{\delta^2 \mathbf{r}_7}{\delta t^2}$ due to deformation is

$$\left. \frac{\delta^2 \mathbf{r}_7}{\delta t^2} \right|_{\text{deformation}} \approx \ddot{\mathbf{A}}_{S7}(t) - \ddot{\mathbf{A}}_{C}(t) - \left[ \ddot{\mathbf{A}}_{S7}(t_{\text{start}}) - \ddot{\mathbf{A}}_{C}(t_{\text{start}}) \right]$$  \hspace{1cm} (16)

Equation (16) may be integrated component by component if both sides are written in terms of unit vectors fixed in CXYZ. Figure 8 shows the results for the roof rail/B-pillar
deflection/crush using this method. Thus, the direct integration of acceleration data provides meaningful results, if the data is chosen properly.

The methodology described here can be used to obtain estimates of the dynamic motion when good estimates of the angular velocity of the vehicle are known from angular velocity transducers. Even without angular velocity data, judicious use of the accelerometer data by subtracting out more constant terms due to angular velocity allows one to estimate dynamic crush. The estimates of 9 inches in Y-dynamic deformation and -3.5 inches in Z-dynamic deformation shown in Figure 8 are based on integrating the differential accelerations of the B-pillar over 200 ms. As shown in Figure 9, the integration of the differential accelerations starting at 500 ms actually produced a larger Y-dynamic deformation result of 10.5 inches and a slightly smaller magnitude negative Z-value of about -2 inches. These estimates compared well to the photogrammetric measurement of lateral roof deformation from the test video. Using the shorter period of time when the B-pillar was experiencing very high acceleration probably yields the better estimate. Since the Z-deformation is small, it does not
appear the sensor rotated much with respect to the vehicle-fixed coordinate system during the time of interest.

![Graph of Y- and Z-deformations of the B-pillar – Integration Start at 500 ms (Autoliv Test B190043)](image)

**Figure 9**

Y- and Z-deformations of the B-pillar – Integration Start at 500 ms
(Autoliv Test B190043)

Of course, the data obtained in this manner is not the total crush time history, but it is a snapshot of the change in the deformation during a small time interval. Since we are concerned with the relative motion of the parts of the vehicle, particularly with respect to restrained occupants, such results are very important.
References for Appendix 1


Mechanical Simulation Corporation, see www.carsim.com.


APPENDIX 2
Driver Lower Neck $M_x$ vs. Lateral Acceleration of the Driver Rail at B-pillar

"Ringing" due to first wheel-to-ground contact

Objective Roof/Pillar Deformation
Roof rail intrudes inboard into driver's survival space

T=497ms

Max peak neck moment, $M_x$, (right ear to right shoulder) resulting from roof/pillar intrusion

T=537ms; 106.4 Nm

Local peak neck $M_x$ due to head contacting roof with roof on ground (-11 to -18 Nm)

Footnote:
Ford did not report time of driver's side roof/pillar deformation in SUV Test 1 (B190042) to NHTSA (3/5/04), NHTSA-1999-5572-75
"Ringing" due to first wheel-to-ground

Max peak neck moment, $M_y$,
(chin-to-chest) resulting from roof pillar intrusion

$T=533\text{ms}; 58.3 \text{ N-m}$

Local peak neck $M_y$ due to head contacting roof with roof on ground

Footnote: Ford did not report time of driver's side roof/ pillar deformation in SUV Test 1 (B190042) to NHTSA (3/5/04), NHTSA-1999-5572-75
Driver Upper Neck $F_z$ vs. Vertical Acceleration of the Driver Rail at B-pillar

"Ringing" due to first wheel-to-ground contact

Local peak neck compression due to head contacting roof with roof on ground (-200 N)

T=497ms
Objective Roof/Pillar Deformation (First peak downward roof acceleration toward dummy)

T=533ms; -958 N
Max Peak Compressive Neck Load, $F_z$

Footnote:
Ford did not report time of driver's side roof/pillar deformation in SUV Test 1 (B190042) to NHTSA (3/5/04), NHTSA-1999-5572-75
Passenger Lower Neck $M_x$ vs. Lateral Acceleration of the Passenger Rail at B-pillar

- Max peak moment load, $M_x$, (left ear to left shoulder) resulting from roof/pillar intrusion: $T=783$ ms; 68 Nm
- Sustained period of deformation into passenger survival space as deformation wave moves from driver's side roof rail to passenger's side: $T=802$ ms
- Local peak neck $M_x$ due to head contacting roof with roof on ground: $T=589$ ms; 9 Nm

"Ringing" due to first wheel-to-ground contact:

"Observable" Roof/Pillar Deformation (source: Frod's presentation to NHTSA, 3/5/03, NHTSA-1999-5572-75)
Passenger Lower Neck $M_y$ vs. Vertical Acceleration of the Passenger Rail at B-pillar

- Sustained period of roof intrusion into passenger survival space as deformation wave moves from driver's side to passenger's side.
- Local peak neck $M_y$, resulting from head contacting roof with roof on ground (12-22 Nm).
- "Ringing" due to first wheel-to-ground contact.
- "Observable" Roof/Pillar Deformation (Source: Ford's presentation to NHTSA 3/5/04, NHTSA-1999-5572-75).
- Objective Roof Deformation Occurs Simultaneous with Peak Neck Moment, $M_y$ (chin toward sternum) resulting from roof/pillar intrusion.

T=729ms; 304.4 Nm
Max Peak Neck Moment, $M_y$ (chin toward sternum) resulting from roof/pillar intrusion.

T=730 ms
Objective Roof Deformation Occurs Simultaneous with Peak Neck Moment, $M_y$.

T=802ms
"Observable" Roof/Pillar Deformation (Source: Ford's presentation to NHTSA 3/5/04, NHTSA-1999-5572-75).
"Ringing" due to first wheel-to-ground contact

Sustained period of deformation into passenger survival space as deformation wave moves from driver's side roof rail to passenger's side

Local peak neck compression due to head contacting roof with roof on ground (-361 N)

T=730ms
Local Peak Roof/Pillar Deformation

T=730ms; -5933 N
Max Peak Compressive Neck Load $F_z$

T=802ms
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)
Objective Roof/Pillar Deformation, Roof intrudes inboard into driver's survival space

"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75

T=513 ms; 43.7 G's

T=590ms

"Ringing" due to first wheel-to-ground contact

T=548ms, -124 N-m
Max Peak Neck Moment, M_x, (right ear to right shoulder) resulting from roof/pillar intrusion
Driver Lower Neck $M_y$ vs. Vertical Acceleration of the Driver Rail at B-pillar

- **Moment $M_y$ (N-m) - Filtered**
  - T=541 ms; 110 N-m
  - Max peak Neck Moment, $M_y$, (chin to sternum) resulting from

- **Vertical Acceleration (G) - Filtered and Transformed**
  - "Ringing" due to first wheel-to-ground

- **Objective Roof/Pillar Deformation**
  - T=513ms
  - "Observable" Roof/Pillar Deformation (first peak downward roof acceleration toward dummy)

- **Max peak Neck Moment, $M_y$**
  - T=590ms
  - "Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)

- **Local peak neck $M_y$ due to head contacting roof with roof on ground**
  - T=828ms; 11 Nm
Driver Upper Neck $F_z$ vs. Vertical Acceleration of the Driver Rail at B-pillar

"Ringing" due to first wheel-to-ground contact

"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)

Objective Roof/Pillar Deformation (first peak downward roof acceleration toward dummy)

T=540ms; -1960 N Max Peak Compressive Neck Load, $F_z$

T=506ms Peak Compressive Neck Load, $F_z$

T=590ms "Observable" Roof/Pillar Deformation

T=621ms; -295 N Local peak neck compression due to head contacting roof with roof on ground

T=513ms Objective Roof/Pillar Deformation (first peak downward roof acceleration toward dummy)
Passenger Lower Neck $M_x$ vs. Lateral Acceleration of the Passenger Rail at B-pillar

- "Ringing" due to first wheel-to-ground contact
- Sustained period of deformation into passenger survival space as deformation wave moves from driver's side roof rail to passenger's side
- $T=862 \text{ms}$: "Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/03, NHTSA-1999-5572-75)
- $T=774 \text{ms}$: 97.7 N-m Max peak moment load, $M_x$, (left ear to left shoulder) resulting from roof/pillar intrusion
- $T=592 \text{ms}$: 12 N-m Local peak neck $M_x$ due to head contacting roof with roof on ground
Passenger Lower Neck $M_y$ vs. Vertical Acceleration of the Passenger Rail at B-pillar

T=764ms; 177 Nm
Max Peak Moment, $M_y$, (chin to sternum) resulting from roof/pillar intrusion

T=862ms
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/03, NHTSA-1999-5572-75)

"Ringing" due to first wheel-to-ground contact

Sustained period of roof deformation into passenger survival space as deformation wave moves from driver's side roof rail to passenger's side

Local peak neck $M_y$ due to head contacting roof with roof on ground (20 - 24 Nm)
Passenger Upper Neck Fz vs. Vertical Acceleration of the Passenger Rail at B-pillar

"Ringing" due to first wheel-to-ground contact

Sustained period of roof deformation into passenger survival space as deformation wave moves from driver's side roof rail to passenger's side

T=764 ms; -3245 N
Peak Compressive Neck Load, Fz

T=509; -50 N
Local peak neck compression due to head contacting roof with roof on ground

T=862 ms
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)

Vertical Acceleration (G) - Filtered and Transformed

Force Fz (N) - Filtered
T=494 ms
Objective Roof/Pillar Deformation
Roof rail intrudes inboard into driver's survival space

T=540 ms; -167 Nm
Max peak neck moment, M_x, (right ear to right shoulder) resulting from roof/pillar intrusion (Ford reported peak at 515 ms)

Footnote:
Ford did not report time of driver's side roof/pillar deformation in SUV Test 4 (B180220) to NHTSA (3/5/04), NHTSA-1999-5572-75
Driver Lower Neck $M_y$ vs. Vertical Acceleration of the Driver Rail at B-pillar

- Max peak neck moment, $M_y$, (chin-to-chest) resulting from roof/pillar intrusion

Footnote: Ford did not report time of driver's side roof/pillar deformation in SUV Test 4 (B180220) to NHTSA (3/5/04), NHTSA-1999-5572-75
Driver Upper Neck $F_z$ vs. Vertical Acceleration of the Driver Rail at B-pillar

Footnote:
Ford did not report time of driver's side roof/pillar deformation in SUV Test 4 (B180220) to NHTSA (3/5/04), NHTSA-1999-5572-75
Passenger Lower Neck $M_x$ vs. Lateral Acceleration of the Passenger Rail at B-pillar

T=760 ms; 41 Nm
Max Peak Neck Moment, $M_x$ (left ear to left shoulder) resulting from roof/pillar intrusion

T=747ms
Objective Roof/Pillar Deformation (Roof rail intrudes inboard into passenger survival space

T=800ms
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)
Passenger Lower Neck $M_y$ vs. Vertical Acceleration of the Passenger Rail at B-pillar

$T=800\text{ms}$
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)

$T=742\text{ ms}$
Objective Roof/Pillar Deformation (local peak downward roof acceleration toward dummy)

$T=751\text{ ms}$; $261\text{ Nm}$
Max peak neck moment, $M_y$ (chin to chest) resulting from roof/pillar intrusion.

$T=536\text{ ms}$; $10\text{ Nm}$
Local peak neck $M_y$ due to head contacting roof with roof on ground.
Passenger Upper Neck $F_z$ vs. Vertical Acceleration of the Passenger Rail at B-pillar

Compressive neck loads due to head contacting roof (200-260 N)

T=800ms
"Observable" Roof/Pillar Deformation (source: Ford's presentation to NHTSA, 3/5/04, NHTSA-1999-5572-75)
APPENDIX 3

PROBLEM:

The domain of observation is a one-second time span subdivided into 20,000 time increments of .00005 sec. each. At each time increment, various measures of acceleration, force, and moments are recorded for a number of experiments.

Let $T_{\text{max}}$ be the time (in milliseconds) at which the maximum value of a measure is observed in an experiment. For example, in Test B190042, the Driver-Side Peak Force was detected at 533 ms. In Test B190043, this peak was found at 540 ms.

So $T_{\text{max}}(1) = 533$ and $T_{\text{max}}(2) = 540$. The absolute difference between these two observations was 7 ms. Was this a coincidence? **What is the probability that the two occurrences would be that close to each other due to random chance alone?**

SOLUTION:

If we assume that in any test, $T_{\text{max}}$ is equally likely to occur at any time $t = 0, \ldots, 1000$ ms, then $t$ belongs to a “uniform” probability distribution, as shown here:

![Uniform Distribution Diagram](image)

If the experiment is done twice, then $\Delta(t) = T_{\text{max}}(1) - T_{\text{max}}(2)$ belongs to a “triangular” distribution, as shown here:
The shaded area (not drawn to scale) represents the probability that the difference between the \( T_{\text{max}} \) results in two different tests will be no more than 7 ms.

By integration, that probability is .014. In other words, there is a 98.6% probability that the difference between these results is NOT due to random chance alone. In the vast majority of statistical studies, this would be interpreted as an observation that was very unlikely to be due to chance alone.

The remainder of this report applies this reasoning to a number of measures from three experimental tests.

TIME OF OCCURRENCE OF ROOF/PILLAR DEFORMATION AND PEAK NECK LOADS (ms)

<table>
<thead>
<tr>
<th>TEST PARAMETER</th>
<th>DRIVER</th>
<th>PASSENGER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B190042</td>
<td>B190043</td>
</tr>
<tr>
<td>Objective Roof/Pillar Deformation</td>
<td>497</td>
<td>513</td>
</tr>
<tr>
<td>Peak ( F_z )</td>
<td>533</td>
<td>540</td>
</tr>
<tr>
<td>Peak ( M_y )</td>
<td>533</td>
<td>541</td>
</tr>
<tr>
<td>Peak ( M_x )</td>
<td>537</td>
<td>548</td>
</tr>
</tbody>
</table>
COMPARISONS:

In making test-to-test pairwise comparisons of any result above, the general formula for the probability that the observed difference is NOT due to chance alone is given by:

\[
\text{Significance Probability} = 1 - 0.002 |\delta-t| + 0.000001 (\delta-t)^2
\]

<table>
<thead>
<tr>
<th>TEST PARAMETER</th>
<th>DRIVER 1</th>
<th>DRIVER 2</th>
<th>(\delta-t)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak (F_z)</td>
<td>533</td>
<td>540</td>
<td>-7</td>
<td>98.6%</td>
</tr>
<tr>
<td>Peak (M_y)</td>
<td>533</td>
<td>541</td>
<td>-8</td>
<td>98.4%</td>
</tr>
<tr>
<td>Peak (M_x)</td>
<td>537</td>
<td>548</td>
<td>-11</td>
<td>97.8%</td>
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</table>

<table>
<thead>
<tr>
<th>TEST PARAMETER</th>
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<th>DRIVER 2</th>
<th>(\delta-t)</th>
<th>Probability</th>
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</thead>
<tbody>
<tr>
<td>Peak (F_z)</td>
<td>533</td>
<td>516</td>
<td>17</td>
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<tr>
<td>Peak (M_y)</td>
<td>533</td>
<td>517</td>
<td>16</td>
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</tr>
<tr>
<td>Peak (M_x)</td>
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<td>540</td>
<td>-3</td>
<td>99.4%</td>
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<table>
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<tr>
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<th>Probability</th>
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<tbody>
<tr>
<td>Peak (F_z)</td>
<td>540</td>
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<td>24</td>
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<tr>
<td>Peak (M_y)</td>
<td>541</td>
<td>517</td>
<td>24</td>
<td>95.3%</td>
</tr>
<tr>
<td>Peak (M_x)</td>
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<td>540</td>
<td>8</td>
<td>98.4%</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST PARAMETER</th>
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<th>DRIVER 1</th>
<th>DRIVER 2</th>
<th>(\delta-t)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak (F_z)</td>
<td>730</td>
<td>764</td>
<td>-34</td>
<td>93.3%</td>
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</tr>
<tr>
<td>Peak (M_y)</td>
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<td>764</td>
<td>-35</td>
<td>93.1%</td>
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<tr>
<td>Peak (M_x)</td>
<td>783</td>
<td>774</td>
<td>9</td>
<td>98.2%</td>
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</tr>
<tr>
<td>TEST PARAMETER</td>
<td>PASSENGER</td>
<td>Significance</td>
<td>Probability</td>
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<td>-----------</td>
<td>--------------</td>
<td>-------------</td>
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<tr>
<td>Peak $F_z$</td>
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<td>743</td>
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<tr>
<td>Peak $M_y$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST PARAMETER</th>
<th>PASSENGER</th>
<th>Significance</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
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<td>764</td>
<td>743</td>
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<tr>
<td>Peak $M_y$</td>
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<td>760</td>
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</tbody>
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